Neural Networks

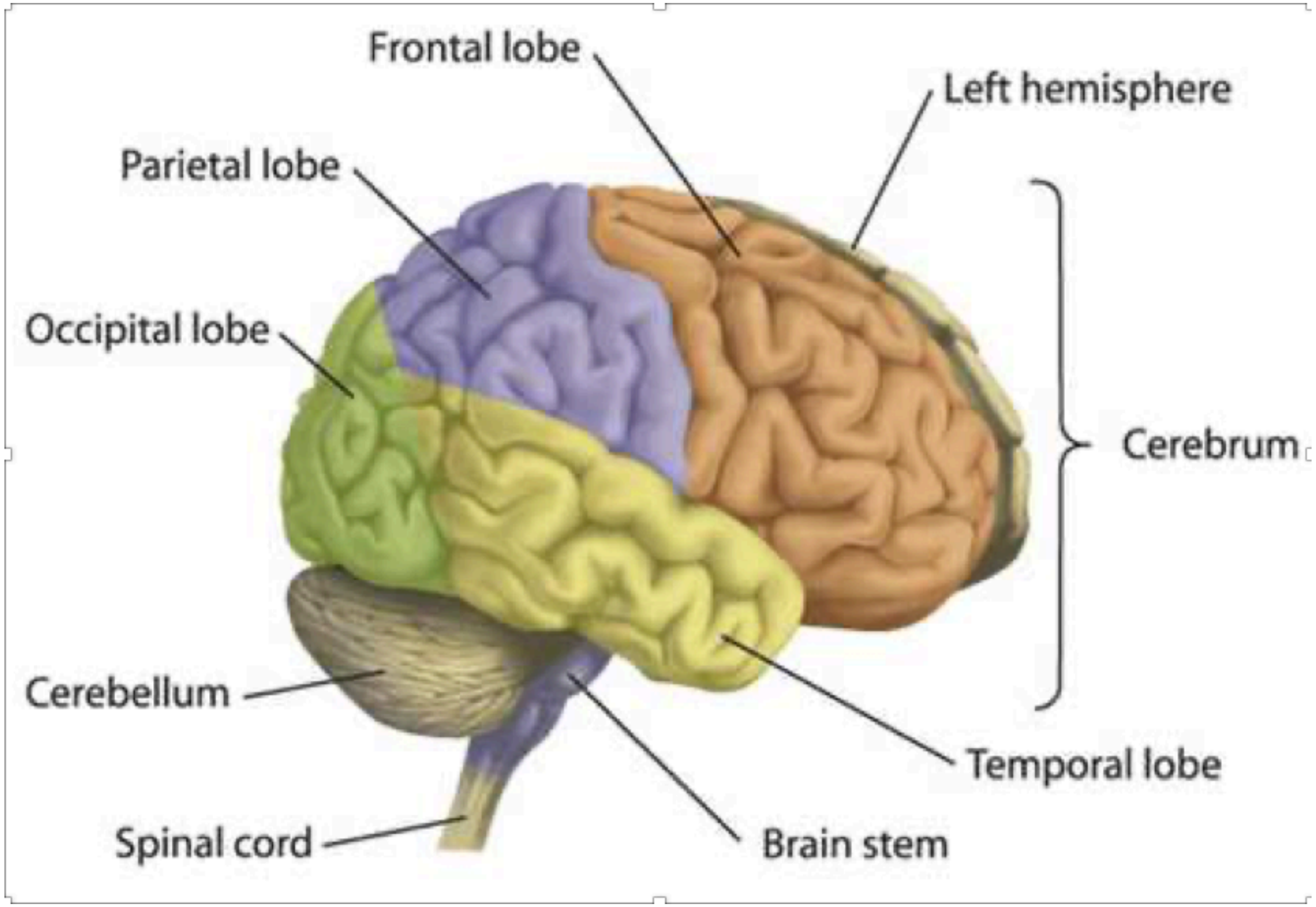
COMP3411/9814: Artificial Intelligence

Lecture Overview

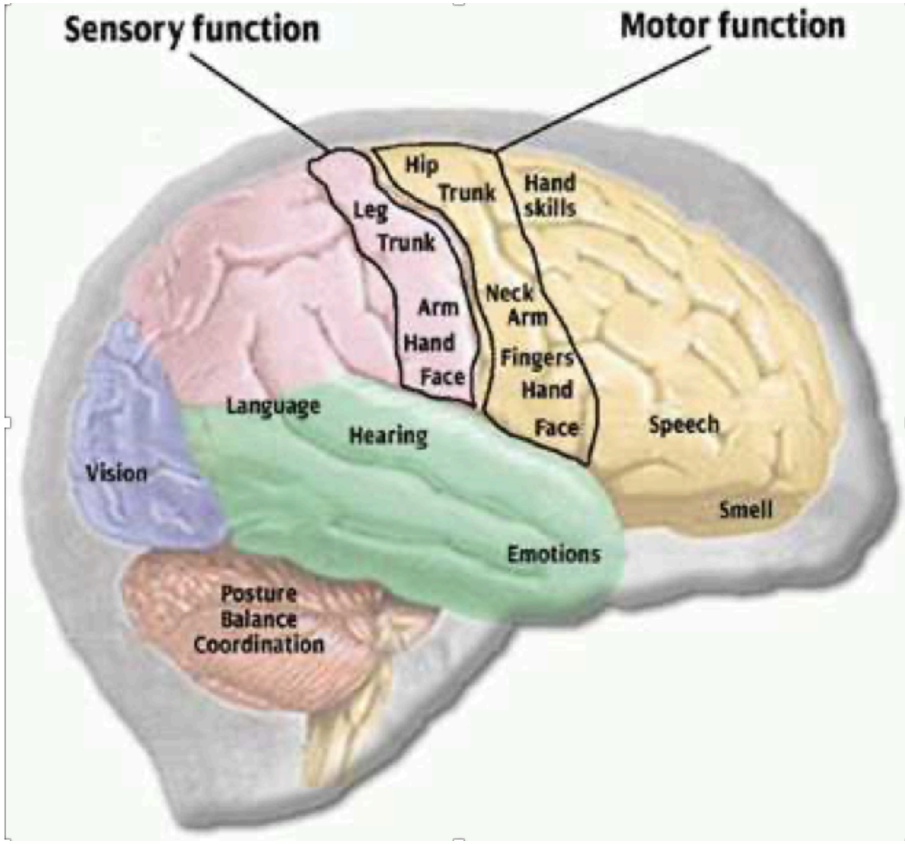
- Neurons – Biological and Artificial
- Perceptron Learning
- Linear Separability
- Multi-Layer Networks
- Back propagation
- Applications of neural networks

Sub-Symbolic Processing

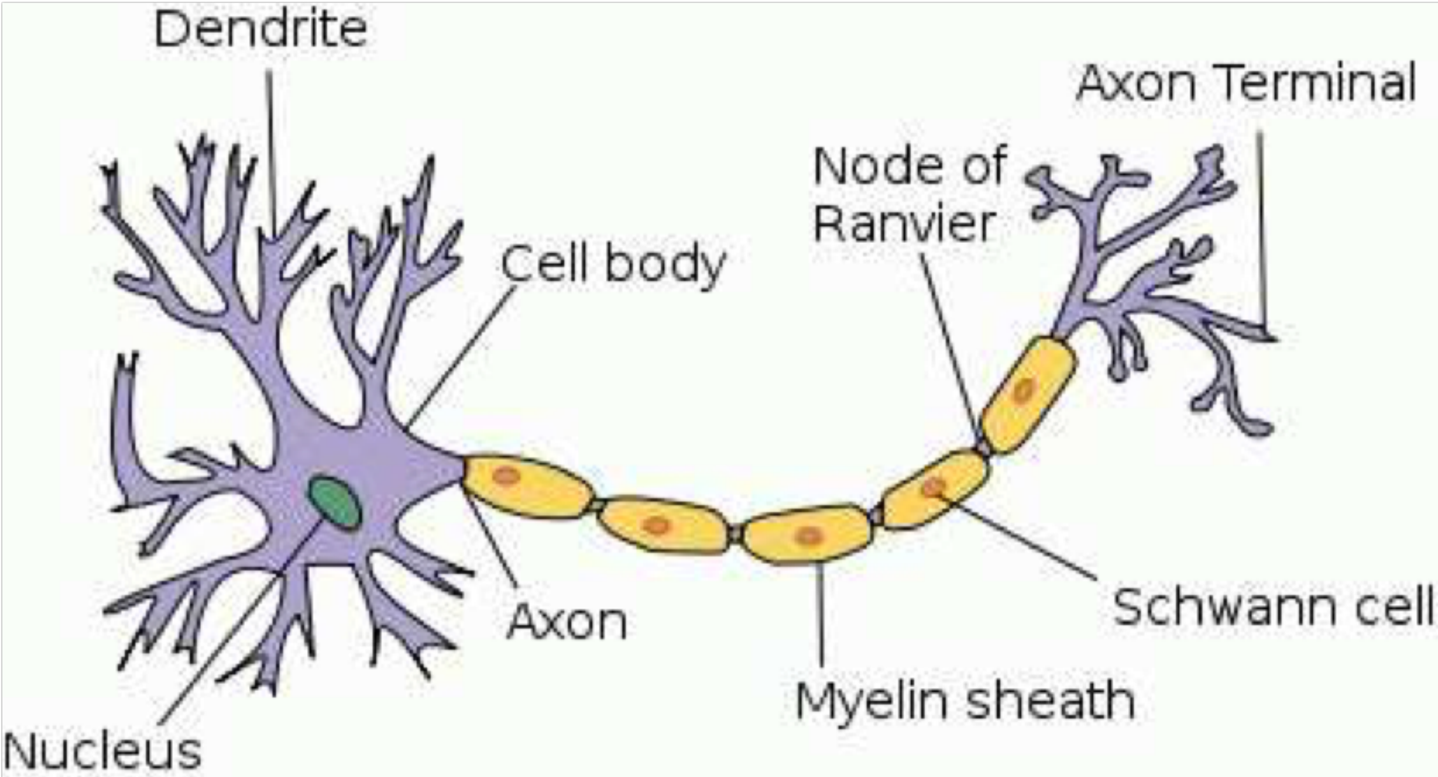
Brain Regions



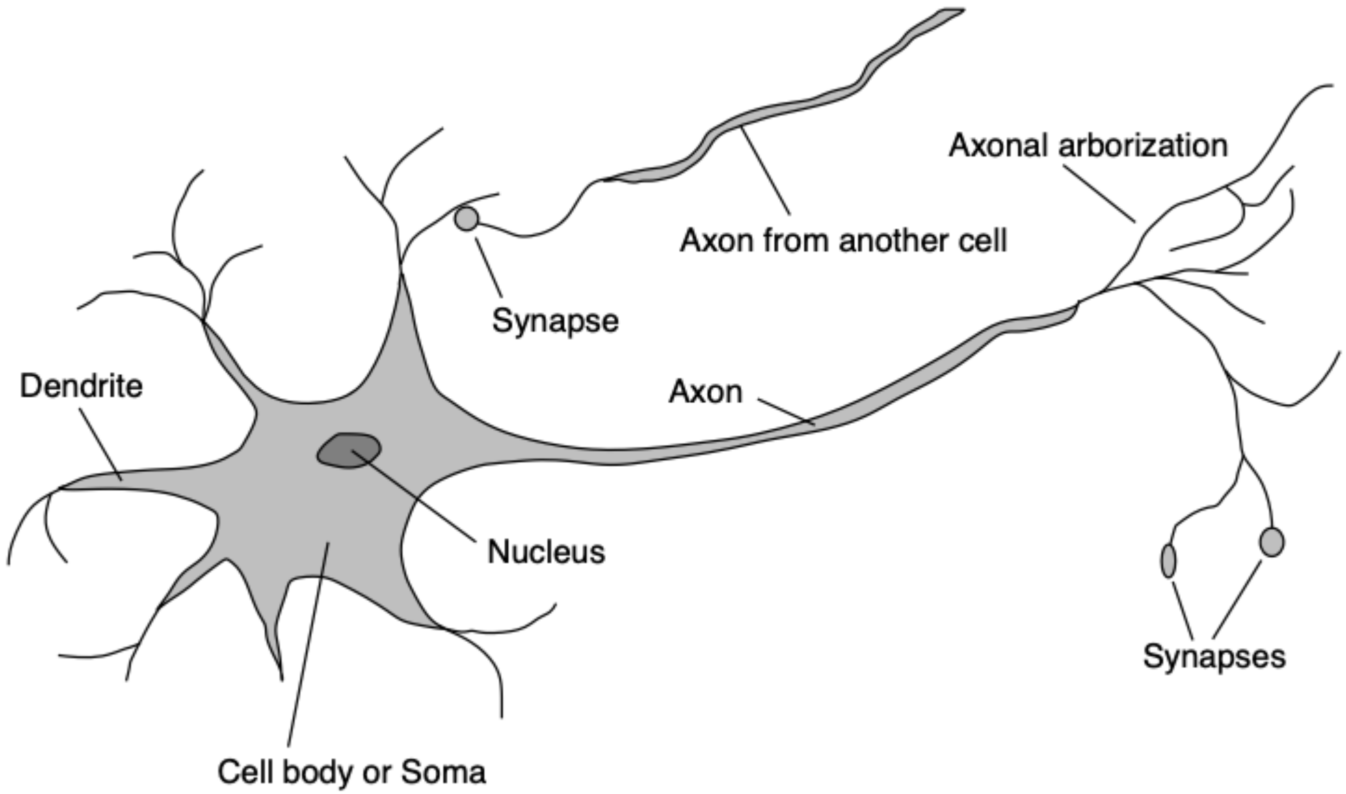
Brain Regions



Structure of a Typical Neuron



Brains


Axonal arborization
Axon from another cell
Synapse
Dendrite
Axon
Nucleus
Synapses
Cell body or Soma

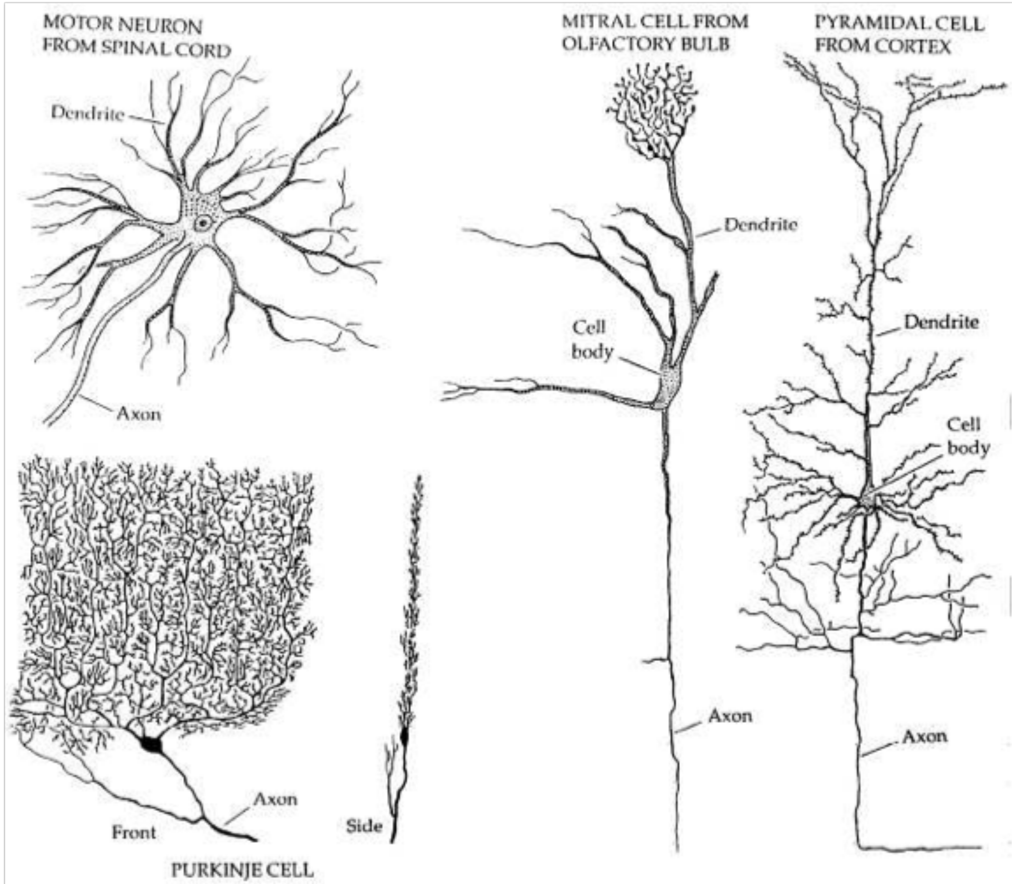
Biological Neurons

- The brain is made up of neurons (nerve cells) which have
  - a cell body (soma)
  - dendrites (inputs)
  - an axon (outputs)
  - synapses (connections between cells)

- Synapses can be excitatory or inhibitory and may change over time.

- When the inputs reach some threshold an action potential (electrical pulse) is sent along the axon to the outputs.

Variety of Neuron Types



The Big Picture

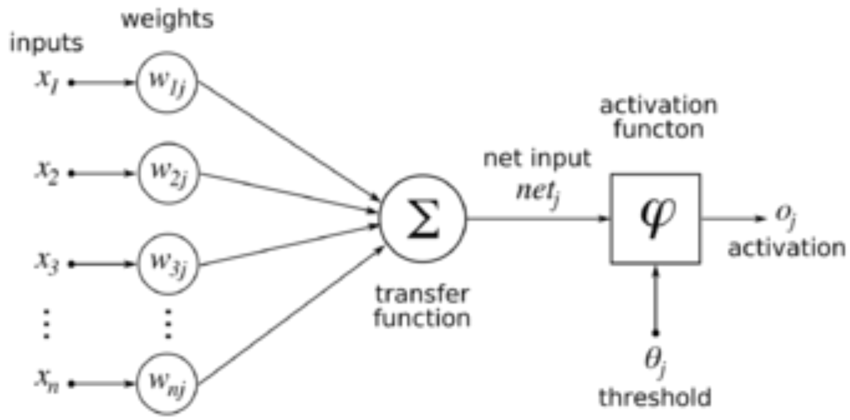
- Human brain has 100 billion neurons with an average of 10,000 synapses each
- Latency is about 3-6 milliseconds
- At most a few hundred “steps” in any mental computation, but massively parallel

Artificial Neural Networks

- Information processing architecture loosely modelling the brain
- Consists of many interconnected processing units (neurons)
  - Work in parallel to accomplish a global task
- Generally used to model relationships between inputs and outputs or to find patterns in data

Artificial Neural Networks (ANN)

- ANNs nodes have
  - inputs edges with some weights
  - outputs edges with weights
  - activation level (function of inputs)



- Weights can be positive or negative and may change over time (learning).
- The input function is the weighted sum of the activation levels of inputs.
- The activation level is a non-linear transfer function ***g*** of this input:

$$\text{activation}_i = g(s_i) = g\left(\sum_j w_{ij} x_j\right)$$

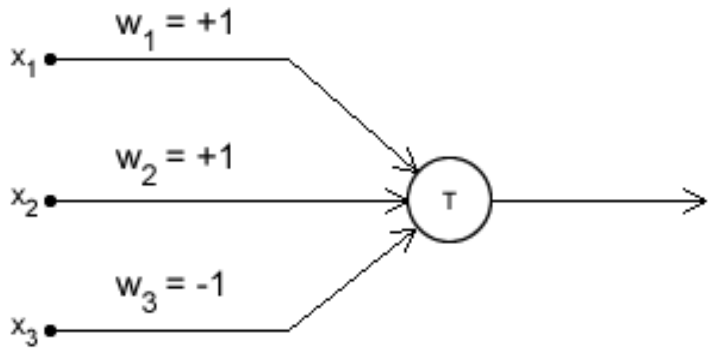
Some nodes are inputs (sensing), some are outputs (action)

First artificial neurons: McCulloch-Pitts (1943)

- McCulloch-Pitts model:
  - Inputs either 0 or 1.
  - Output 0 or 1.
  - Input can be either excitatory or inhibitory.

- Summing inputs
  - If input is 1, and is excitatory, add 1 to sum.
  - If input is 1, and is inhibitory, subtract 1 from sum.

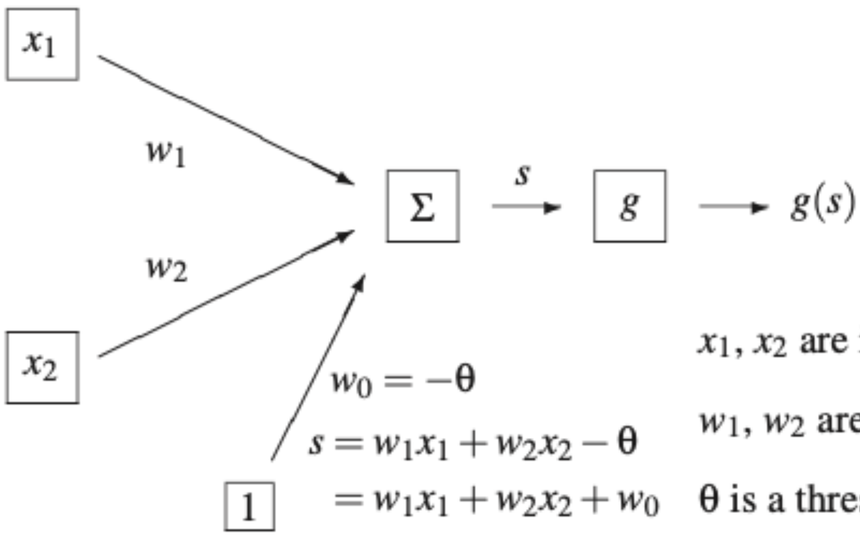
- Threshold,
  - if sum < threshold, T, output 0.
  - Otherwise, output 1.



$$sum = x_1 \bullet w_1 + x_2 \bullet w_2 + x_3 \bullet w_3 + \ldots$$

if *sum* < *T* **then** *output* is 0
else *output* is 1.

McCulloch & Pitts Model of a Single Neuron



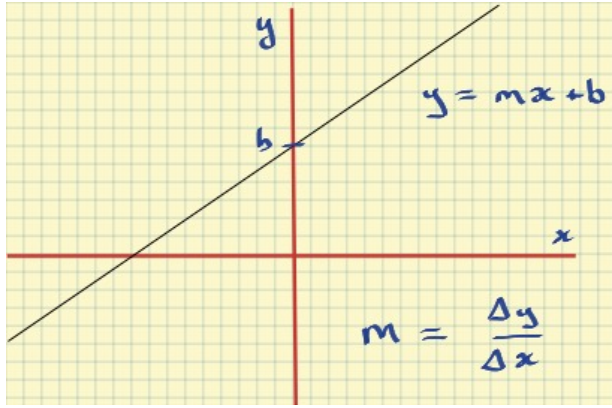
x_1, x_2 are inputs

w_1, w_2 are synaptic weights

θ is a threshold

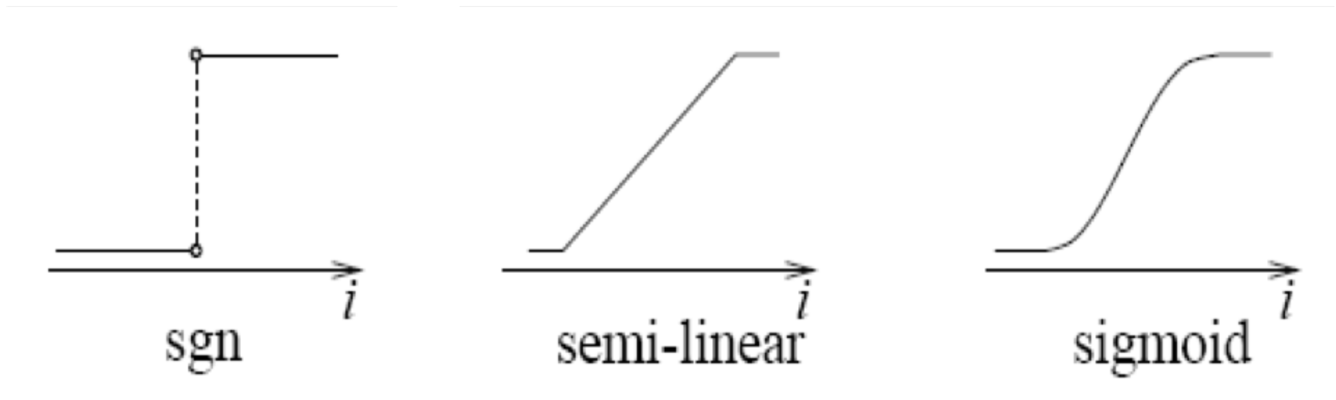
w_0 is a **bias** weight

g is transfer function



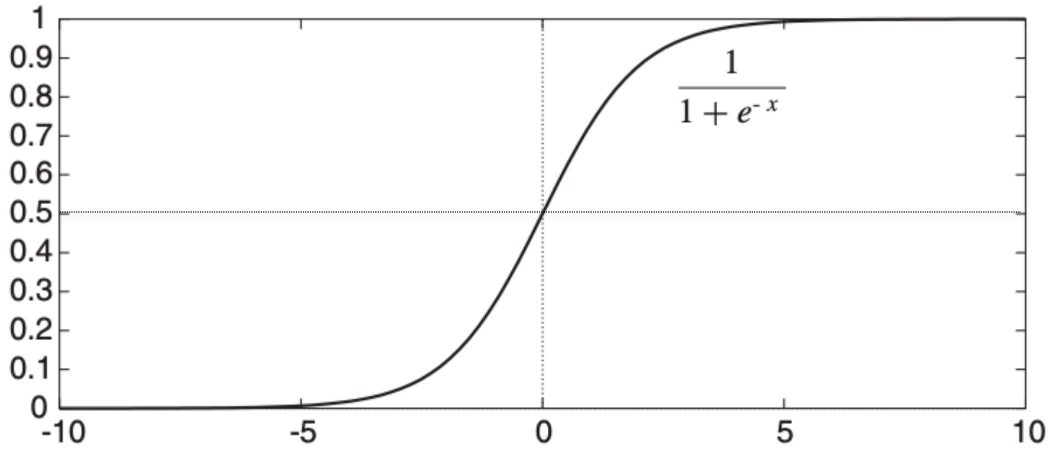
Activation Functions

Function $g(s)$ takes the weighted sum of inputs and produces output for node, given some threshold.



$$g(s) = \begin{cases} 1 \text{ if } s \geq 0 \\ 0 \text{ if } s < 0 \end{cases}$$

The sigmoid or logistic activation function

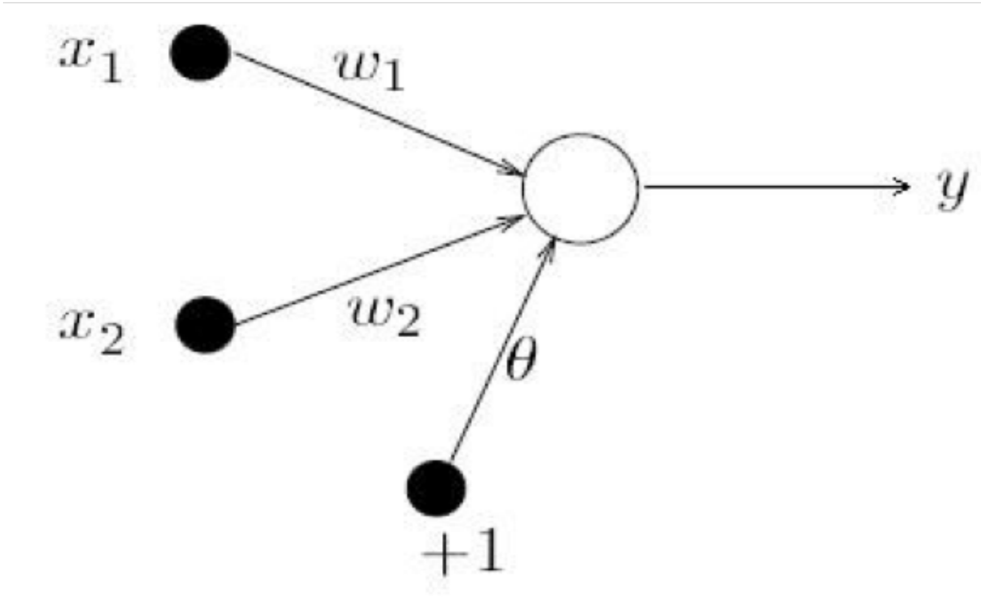


$$f(x) = \frac{1}{1 + e^{-}x}$$

Derivative $f'(x) = f(x)(1 - f(x))$ is the slope of the function

Simple Perceptron

The perceptron is a single layer feed-forward neural network.

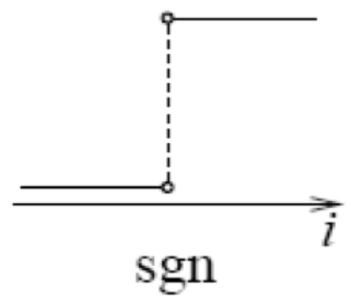


Simple Perceptron

Simplest output function

$$y = \mathrm{sgn}\left(\sum_{i=1}^{2} w_i x_i + \theta\right)$$

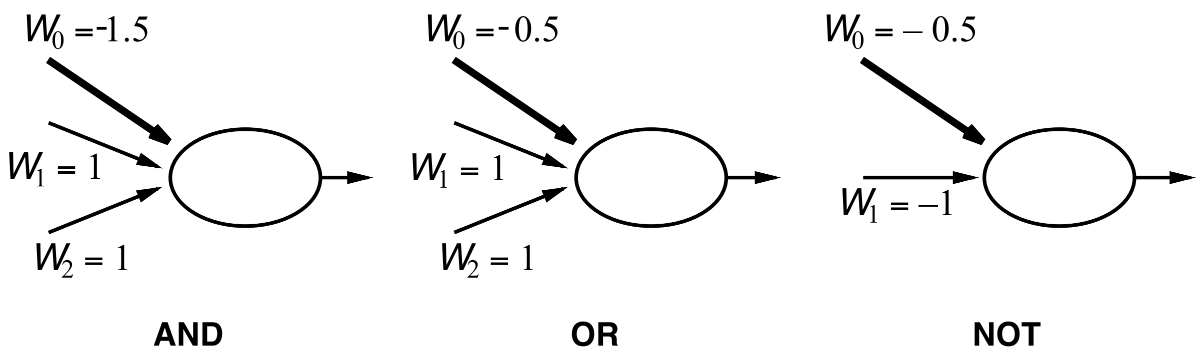
$$\mathrm{sgn}(s) = \begin{cases} 1 & \text{if } s > 0 \\ -1 & \text{otherwise.} \end{cases}$$



Used to classify patterns said to be linearly separable

Implementing logical functions

McCulloch and Pitts - every Boolean function can be implemented:



Linear Separability

Examples:

AND	$w_1 = w_2 = \ \ 1.0, \quad w_0 = -1.5$
OR	$w_1 = w_2 = \ \ 1.0, \quad w_0 = -0.5$
NOR	$w_1 = w_2 = -1.0, \quad w_0 = \ \ 0.5$

Can we train a perceptron net to learn a new function?

Yes, as long as it is linearly separable

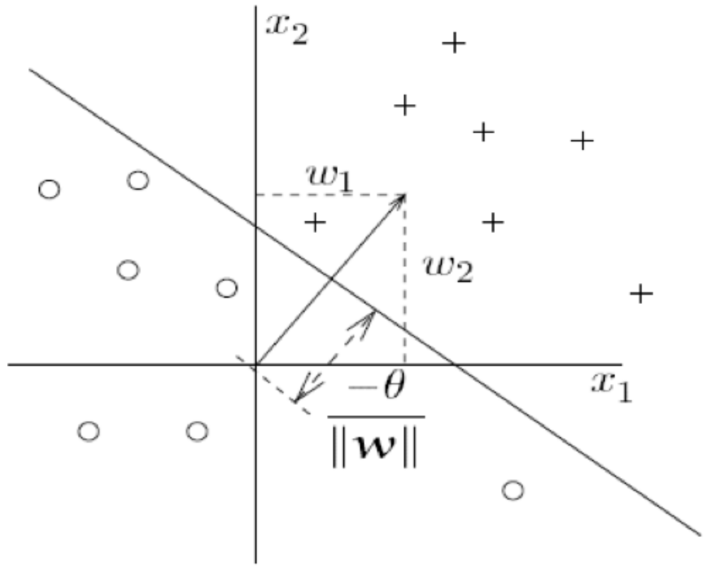
Linear Separability

The bias is proportional to the offset of the plane from the origin

The weights determine the slope of the line

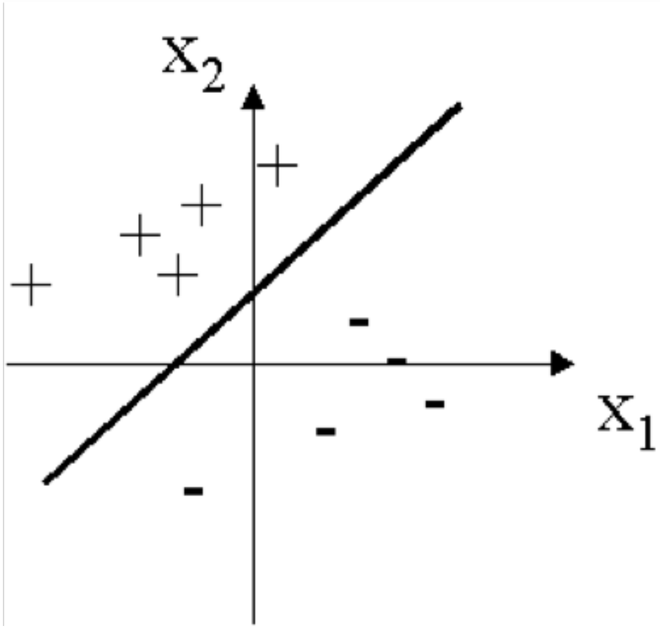
The weight vector is perpendicular to the plane

$$x_2 = -\frac{w_1}{w_2}x_1 - \frac{\theta}{w_2}$$



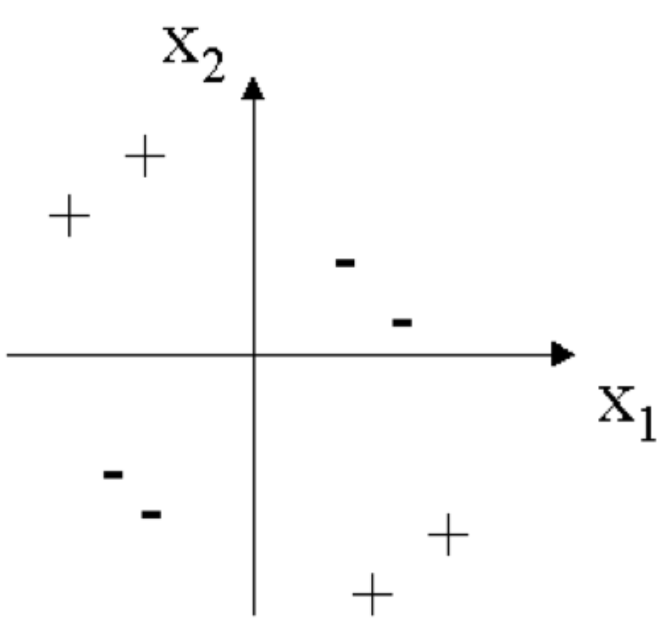
Linear Separability

What kind of functions can a perceptron compute?



Linearly Separable

$$w_1x_1 + w_2x_2 + \theta = 0$$



Not Linearly Separable

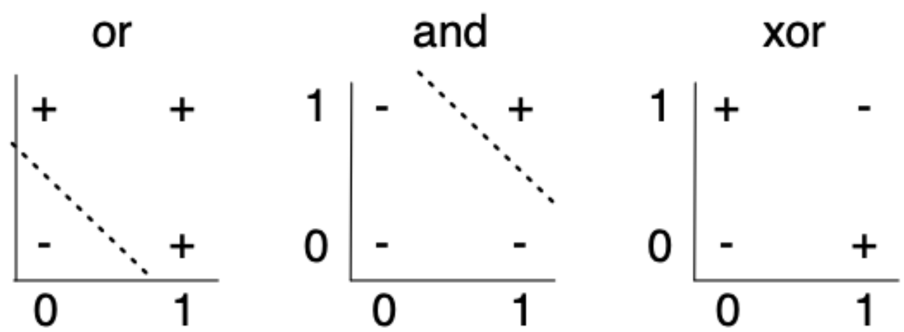
Linearly Separable

- **Linearly separable** if there is a hyperplane where classification is true on one side of hyperplane and false on other side

- For the sigmoid function, when the hyperplane is:

$$x_1 \bullet w_1 + \ldots + x_n \bullet w_n = 0$$

separates the predictions > 0.5 and < 0.5.

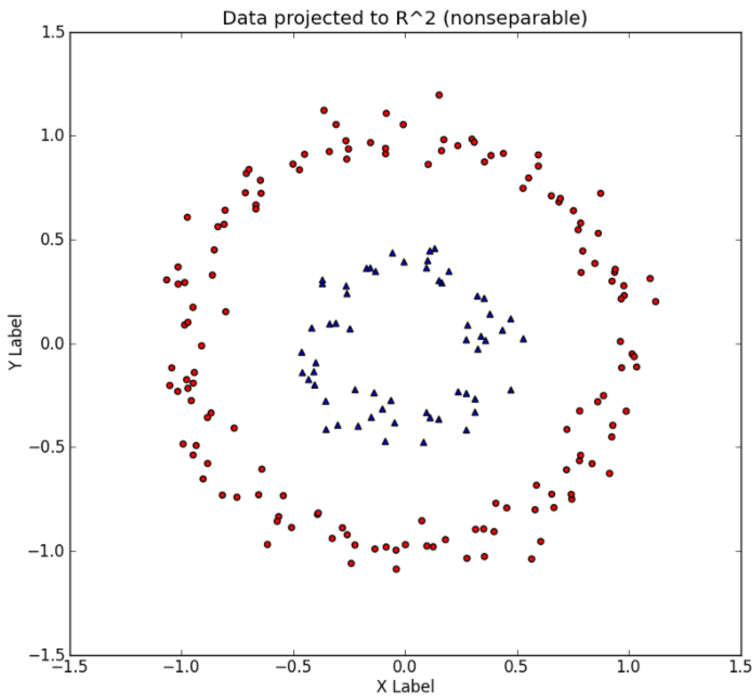


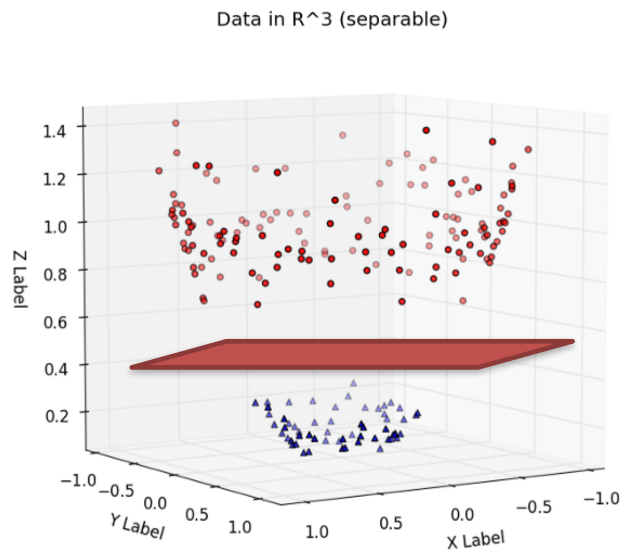
Variants in Linear Separators

- Which linear separator to use can result in various algorithms:
  - Perceptron
  - Logistic Regression
  - Support Vector Machines (SVMs) ...

Kernel Trick

- Project points onto an higher dimensional space
- Becomes linearly separable





Perceptron Learning Algorithm

- Want to train perceptron to classify inputs correctly
- Compare output of network with correct output and adjust the weights and bias to minimise the error
- So learning is parameter optimisation
- Can only handle linearly separable sets

Perceptron Learning Algorithm

- Training set: set of input vectors to train perceptron.
- During training, w_i and θ ***(bias)*** are modified
  - for convenience, let $w_0 = \theta$ and $x_0 = 1$
- η, is the *learning rate*, a small positive number
  - small steps lessen possibility of destroying correct classifications
- Initialise w_i to some values

Perceptron Learning Rule

- Repeat for each training example
  - Adjust the weight, w_i, for each input, x_i.

$$w_i \leftarrow w_i + \eta(d - y) \cdot x_i$$

η > 0 is the learning rate

d is the desired output

y is the actual output

- If output correct, no change
- If d=1 but y=0, w_i is increased when x_i is positive and decreased when x_i is negative (want to increase $\mathbf{w} \bullet \mathbf{x}$)
- If d=0 but y=1, w_i is decreased when x_i is positive and increased when x_i is negative (want to decrease $\mathbf{w} \bullet \mathbf{x}$)

Perceptron Convergence Theorem

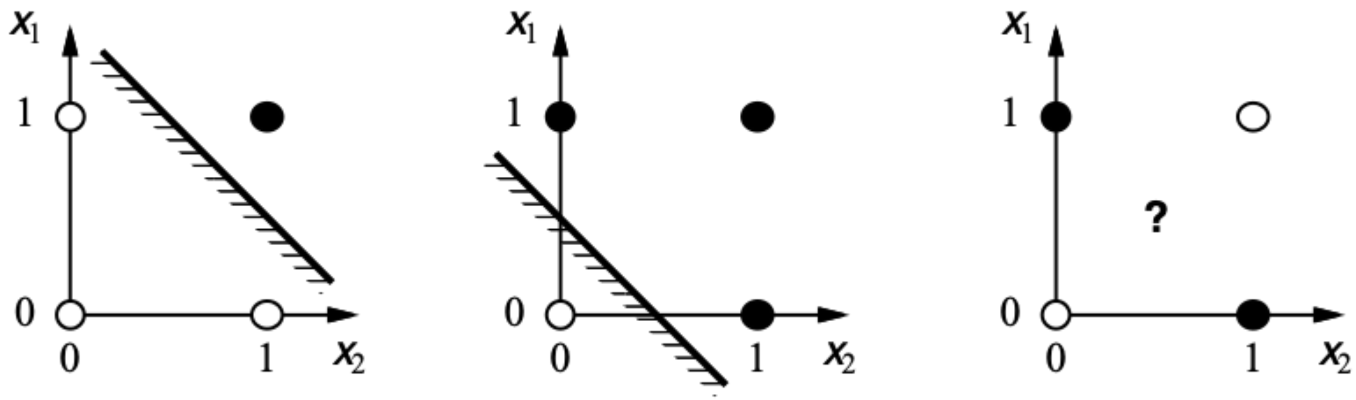
For any data set that is linearly separable, perceptron learning rule is guaranteed to find a solution in a finite number of iterations.

Historical Context

- In 1969, Minsky and Papert published book highlighting limitations of perceptrons
  - Funding agencies redirected funding away from neural network research,
  - preferring instead logic-based methods such as expert systems.
- Known since 1960's that any logical function could be implemented in a 2-layer neural network with step function activations.
- Problem was how to learn weights of multi-layer neural network from training examples
- Solution found in 1976 by Paul Werbos
  - not widely known until rediscovered in 1986 by Rumelhart, Hinton and Williams.

Limitations of Perceptrons

Problem: many useful functions are not linearly separable (e.g. XOR)



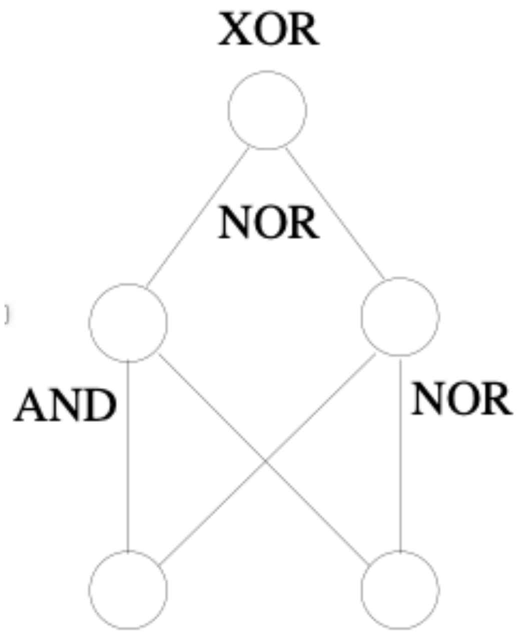
(a) x_1 **and** x_2 (b) x_1 **or** x_2 (c) x_1 **xor** x_2

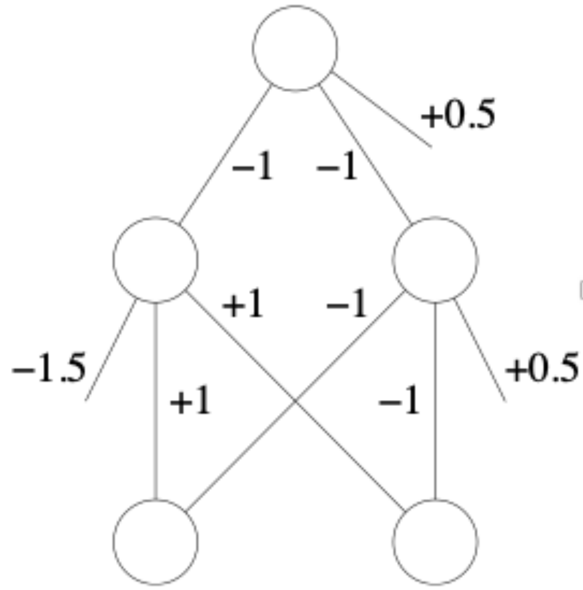
Possible solution:

x1 XOR x2 can be written as: (x1 AND x2) NOR (x1 NOR x2)

Recall that AND, OR and NOR can be implemented by perceptrons.

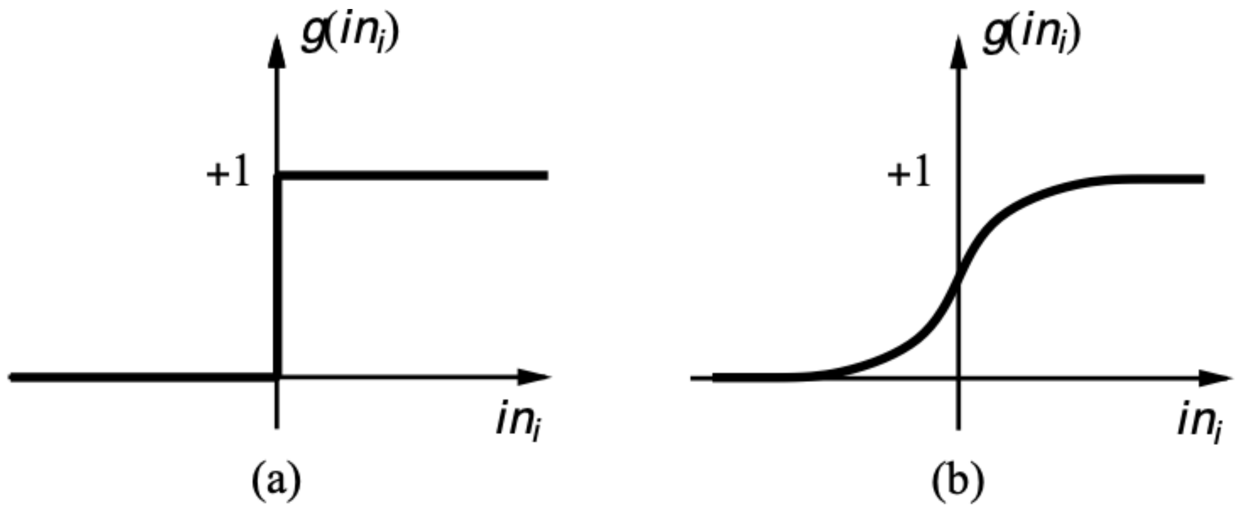
Multi-Layer Neural Networks





- Given an explicit logical function, we can design a multi-layer neural network by hand to compute that function.
- But, if we are just given a set of training data, can we train a multi-layer network to fit these data?

Activation functions

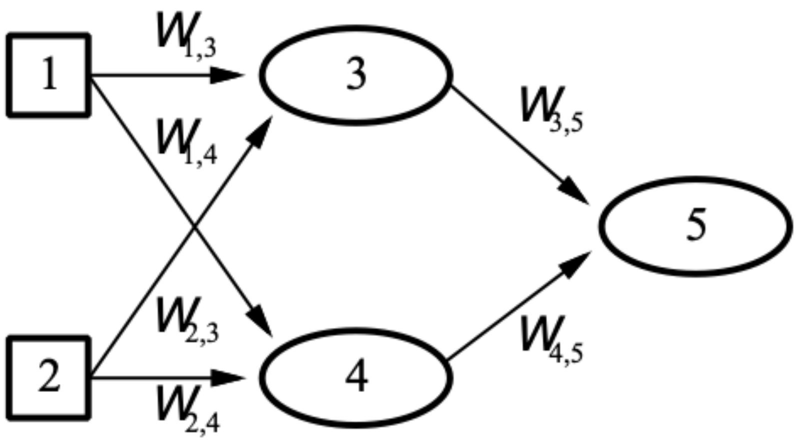


(a) is a step function or threshold function

(b) is a sigmoid function $\dfrac{1}{1 + e^{-x}}$

Changing the bias weight $w_{0,i}$ moves the threshold

Feed-Forward Example



$w_{i,j} \equiv$ weight between node i and node j

- Feed-forward network = a parameterised family of nonlinear functions:

$$\begin{aligned} a_5 &= g\left(W_{3,5} \bullet a_3 + W_{4,5} \bullet a_4\right) \\ &= g\Big(W_{3,5} \bullet g\left(W_{1,3} \bullet a_1 + W_{2,3} \bullet a_2\right) + W_{4,5} \bullet g\left(W_{1,4} \bullet a_1 + W_{2,4} \bullet a_2\right)\Big) \end{aligned}$$

- Adjusting weights changes the function

ANN Training as Cost Minimisation

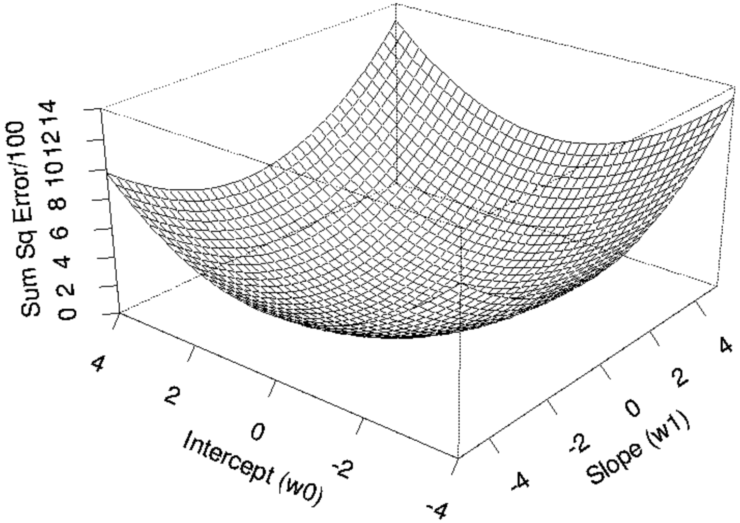
- Define error function Mean Squared Error, E

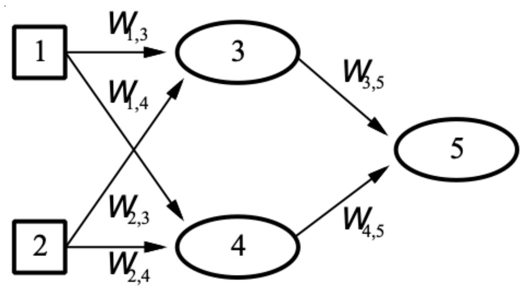
$$E = \frac{1}{2} \sum (d - y)^2$$

y = actual output

d = desired output

- Think of E as height of error landscape of weight space.
- Aim to find a set of weights for which E is very low.



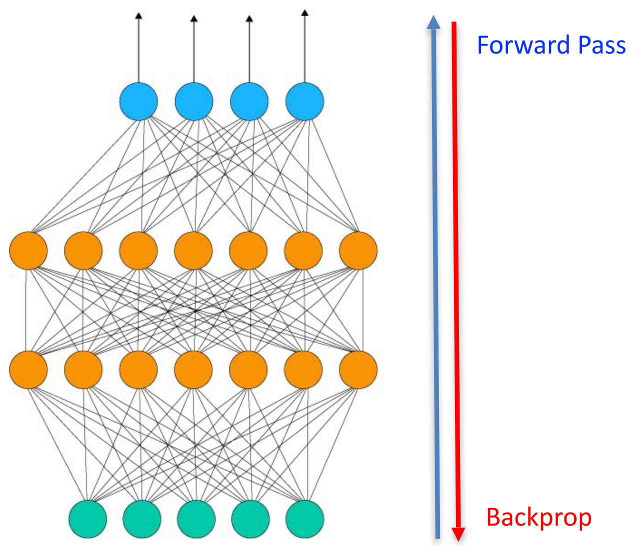


Backpropagation

1. **Forward pass**: apply inputs to “lowest layer” and feed activations forward to get output

2. **Calculate error**: difference between desired output and actual output

3. **Backward pass**: Propagate errors back through network to adjust weights



Gradient Descent

$$E = \frac{1}{2} \sum (d - y)^2$$

If transfer functions are smooth, can use multivariate calculus to adjust weights by taking steepest downhill direction.

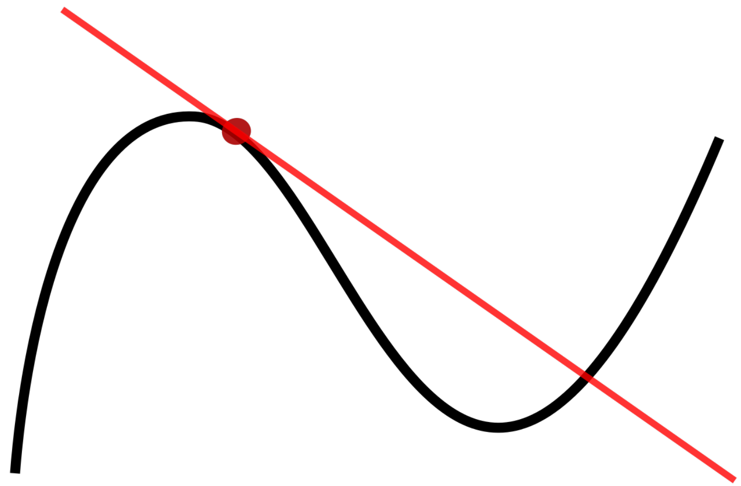
$$w \leftarrow w + \eta \frac{\partial E}{\partial w}$$

Parameter η is the **learning rate**

- How the cost function affects the particular weight
- Find the weight

Derivative of a Function

The derivative of a function is the slope of the tangent at a point



$$y = f(x) = mx + b$$

$$m = \frac{change\ in\ y}{change\ in\ x} = \frac{\Delta\, y}{\Delta\, x}$$

Written $\dfrac{dy}{dx}$

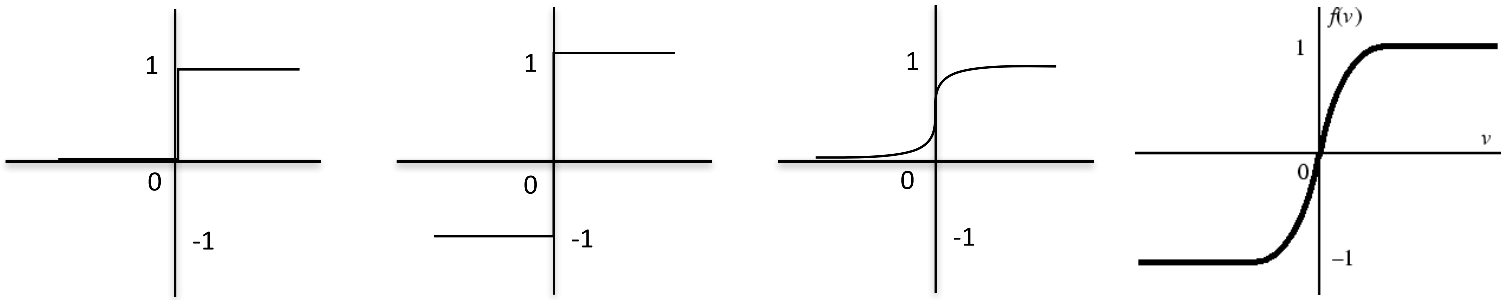
Partial Derivative

Derivative of a function of several variables with respect to one of this variables

If $z = f\left(x,\ y, \ldots\right)$

Derivative with respect to x is written: $\dfrac{\partial z}{\partial x}$

Function must be continuous to be differentiable



Replace the (discontinuous) step function with a differentiable function, such as the sigmoid:

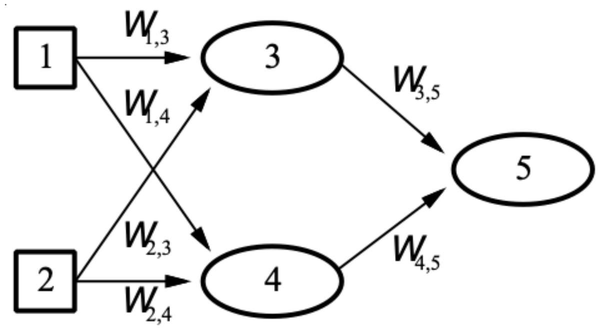
$$g(s) = \frac{1}{1 + e^{-s}}$$

or hyperbolic tangent

$$g(s) = \tanh(s) = \frac{e^{s} - e^{-s}}{e^{s} + e^{-s}} = 2\left(\frac{1}{1 + e^{-s}}\right) - 1 \qquad \text{(-1 to 1)}$$

Chain Rule

$g\left(W_{3,5} \bullet g\left(W_{1,3} \bullet a_1 + W_{2,3} \bullet a_2\right) + W_{4,5} \bullet g\left(W_{1,4} \bullet a_1 + W_{2,4} \bullet a_2\right)\right)$



If $$y = g\big(f(x)\big) \equiv \begin{cases} y = g(u) \\ u = f(x) \end{cases}$$

then $$\frac{\partial y}{\partial x} = \frac{\partial y}{\partial u} \bullet \frac{\partial u}{\partial x}$$

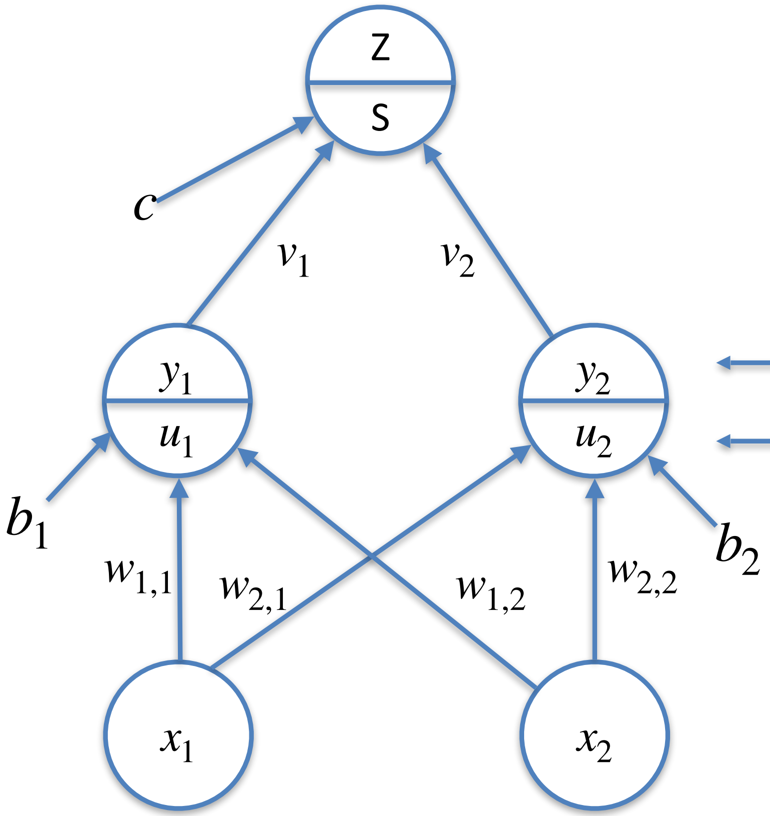
Used chain rule to compute partial derivatives efficiently.

Transfer function must be differentiable (usually sigmoid, or tanh).

Note: if $z(s) = \dfrac{1}{1 + e^{-s}}$ derivative $z'(s) = z(1 - z)$

if $z(s) = \tanh(s)$ derivative $z'(s) = 1 - z^2$

Backpropagation

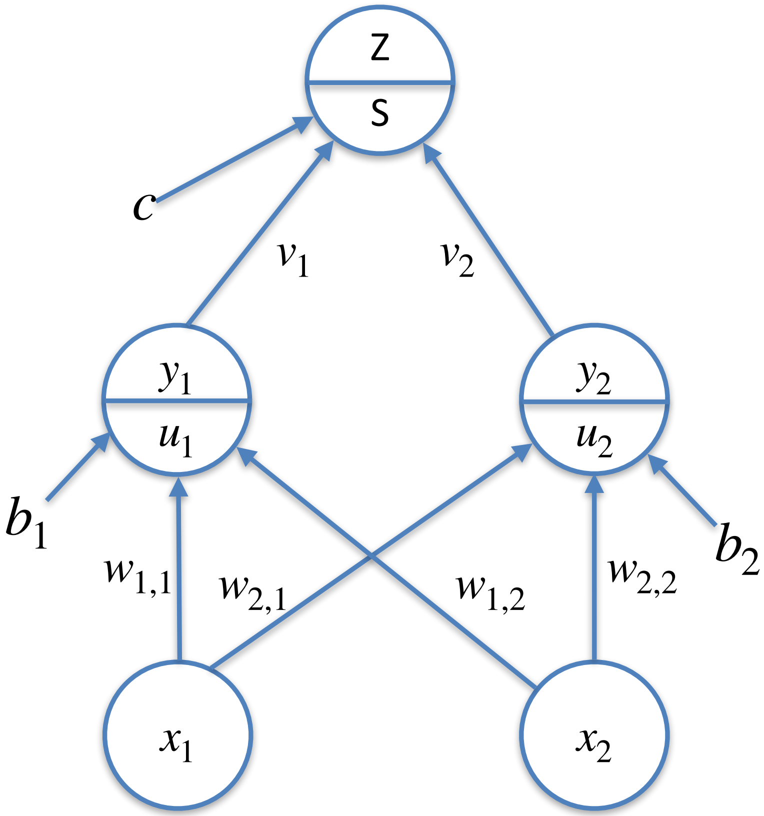


Convenient to split layers into:

sigmoid layer and

linear complete layer

Forward Pass



$$u_1 = b_1 + w_{11}x_1 + w_{12}x_2$$

$$y_1 = g(u_1)$$

$$s = c + v_1 y_1 + v_2 y_2$$

$$z = g(s)$$

$$E = \frac{1}{2}\Sigma(z - t)^2 \qquad (t = target)$$

Backpropagation Algorithm

procedure *BackpropLearner*$(Xs,\ Ys,\ Es,\ layers,\ \eta)$

  repeat

    for each example e **in** Es in random order **do**

      $values[i] \leftarrow Xi(e)$ **for each** input unit i

    for each layer **from** lowest **to** highest **do**

      $values \leftarrow OutputValues(layer, values)$

    $error \leftarrow SumSqError(Ys(e), values)$

    for each $layer$ **from** highest **to** lowest **do**

      $error \leftarrow Backprop(layer, error)$

  until $termination$

Xs: set of input features, $Xs = \{X1, \ldots, Xn\}$

Ys: target features

Es: set of examples from which to learn

$layers$: a sequence of layers

η: learning rate (gradient descent step size)

Backpropagation Algorithm

function g(x) = $1/(1+e^{-x})$

function $SumSqError(Ys, predicted)$ = $[\frac{1}{2}\sum(Ys[j]-predicted[j])^2$ **for each output unit** $j]$

function OutputValues($layer$, $input$) — // $input$ is array with length n_i

  define $input$ $[n]$ to be 1

  $output[j] \leftarrow g\left(\sum_{i=0}^{n} w_{ij} \cdot input[i]\right)$ **for each** j — // *update output for layer*

  return $output$

function Backprop($layer$, $error$) — // *each layer has an input and output array*

  if output layer — // *error is array with length* n_i

    $delta = error$

  else

derivative

    $delta[i] \leftarrow \boxed{output[i] \cdot (1-output[i])} \cdot error[i]$ **for each** i

  $w_{ji} \leftarrow w_{ji} + \eta \cdot delta[j] \cdot input[i]$ **for each** i, j, for learning rate η — // perceptron rule

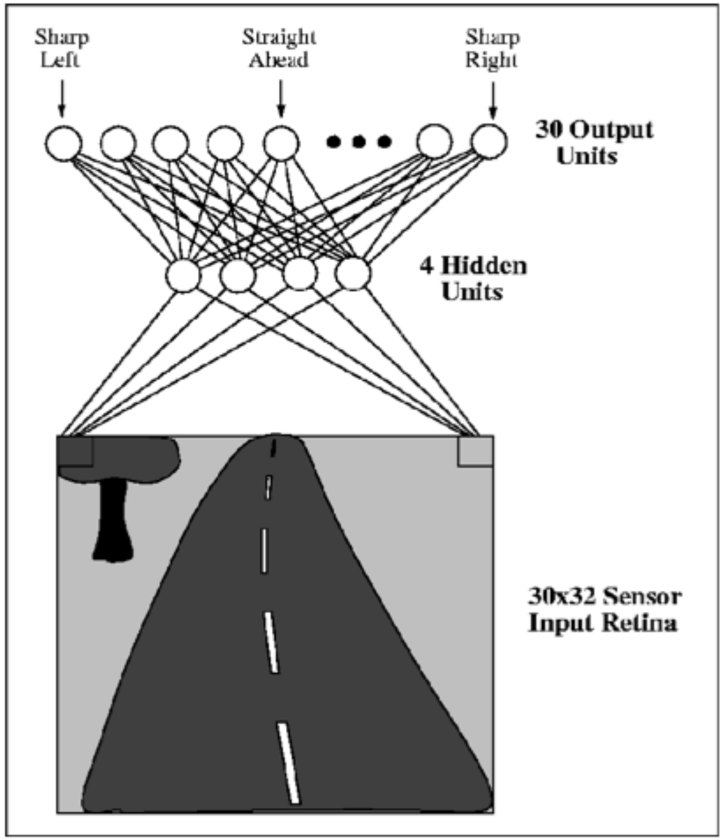
  $delta[i] \leftarrow \sum_j w_{ji} \cdot delta[j]$ **for each** i

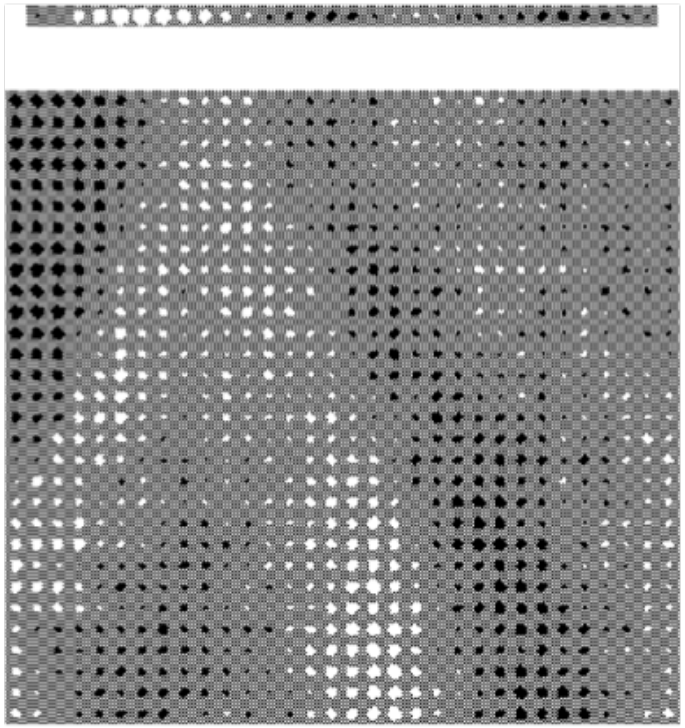
  return $delta$

Neural Network – Applications

- Autonomous Driving
- Game Playing
- Credit Card Fraud Detection
- Handwriting Recognition
- Financial Prediction

ALVINN (First demo of long distance autonomous driving)





ALVINN

- Autonomous Land Vehicle In a Neural Network

- later version included a sonar range finder
  - 8×32 rangefinder input retina
  - 29 hidden units
  - 45 output units

- Supervised Learning, from human actions (Behavioural Cloning)
  - additional “transformed” training items to cover emergency situations

- Drove (mostly) autonomously from coast to coast in USA

Training Tips

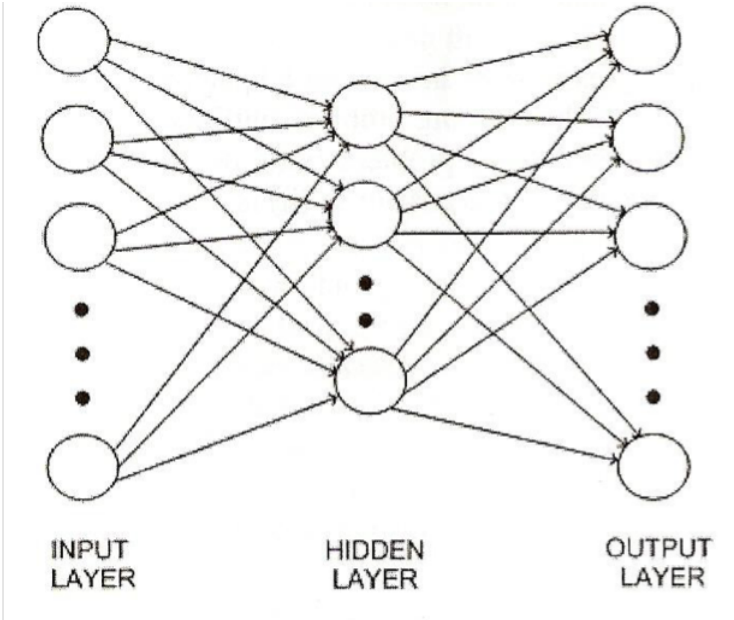
- Re-scale inputs and outputs to be in the range 0 to 1 or –1 to 1
- Initialise weights to very small random values
- On-line or batch learning
- Three different ways to prevent overfitting:
  - limit the number of hidden nodes or connections
  - limit the training time, using a validation set
  - weight decay
- Adjust the parameters: learning rate (and momentum) to suit the particular task

Neural Network Structure

Two main network structures

1. Feed-Forward Network

2. Recurrent Network



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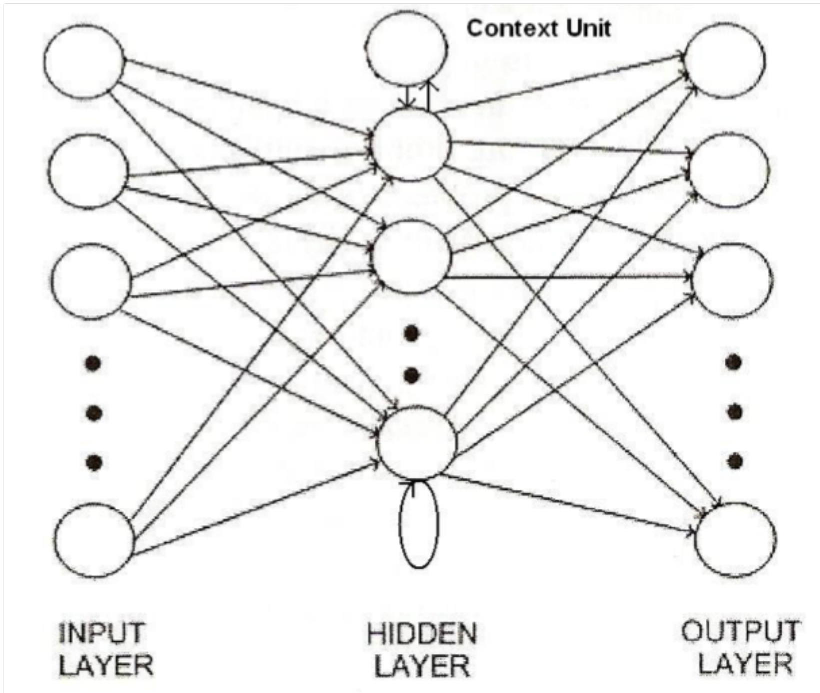

Neural Network Structures

Feed-forward network has connections only in one direction

- Every node receives input from “upstream” nodes; delivers output to “downstream” nodes
  - no loops.
- Represents a function of its current input
  - has no internal state other than the weights themselves.

Recurrent network feeds outputs back into its own inputs

- Activation levels of network form a dynamical system
  - may reach a stable state or exhibit oscillations or even chaotic behaviour
- Response of network to an input depends on its initial state
  - which may depend on previous inputs.
- Can support short-term memory

Neural Networks

- Multiple layers form a hierarchical model, known as **deep learning**
  - **Convolutional neural networks** are specialised for vision tasks
  - **Recurrent neural networks** are used for time series

- Typical real-world network can have 10 to 20 layers with hundreds of millions of weights
  - can take hours, days, months to learn on machines with thousands of cores

Summary

- Vector-valued inputs and outputs
- Multi-layer networks can learn non-linearly separable functions
- Hidden layers learn intermediate representation
  - ✦ How many to use?
- Prediction – Forward propagation
- Gradient descent (Back-propagation)
  - ✦ Local minima problems
- Kernel trick can be introduced through a deep belief network

References

- Poole & Mackworth, Artificial Intelligence: Foundations of Computational Agents, Chapter 7.

- Russell & Norvig, *Artificial Intelligence: a Modern Approach*, Chapters 18.6, 18.7 .